# Unit 2

# Kinematics

## STUDENT'S LEARNING OUTCOMES

**After studying this unit, the students will be able to:**

- describe using examples how objects can be at rest and in motion simultaneously.
- identify different types of motion i.e. translator/, (linear, random, and circular); rotatory and vibratory motions and distinguish among them.
- differentiate with examples between distance and displacement, speed and velocity.
- differentiate with examples between scalar and vector quantities.
- represent vector quantities by drawing.
- define the terms speed, velocity and acceleration.
- plot and interpret distance-time graph and speed-time graph.
- determine and interpret the slope of distance-time and speed-time graph.
- determine from the shape of the graph, the state of a body
  - i. at rest
  - ii. moving with constant speed
  - iii. moving with variable speed.

**This unit is built on**
Force and Motion
- Science-IV

**This unit leads to:**
Motion and force
- Physics-X

**Major Concepts**

2.1 Rest and motion

2.2 Types of motion (Translator/, rotatory, vibratory)

2.3 Terms associated with motion;
- Position
- Distance and displacement
- Speed and velocity
- Acceleration

2.4 Scalars and vectors

2.5 Graphical analysis of motion;
- Distance-time graph
- Speed-timegraph

2.6 Equations of Motion;
- $S = vt$
- $v_f = v_i + at$
- $S = v_i t + \frac{1}{2} at^2$
- $v_f^2 - v_i^2 = 2aS$

2.7 Motion due to gravity

- calculate the area under speed-time graph to determine the distance travelled by the moving body.
- derive equations of motion for a body moving with uniform acceleration in a straight line using graph.
- solve problems related to uniformly accelerated motion using appropriate equations.
- solve problems related to freely falling bodies using 10 $ms^{-2}$ as the acceleration due to gravity.

## INVESTIGATION SKILLS:

The students will be able to:

- demonstrate various types of motion so as to distinguish between translatory, rotatory and vibratory motions.
- measure the average speed of a 100 m sprinter.

## SCIENCE, TECHNOLOGY AND SOCIETY CONNECTION:

The students will be able to:

- list the effects of various means of transportation and their safety issues.
- the use of mathematical slopes (ramps) of graphs or straight lines in real life applications.
- interpret graph from newspaper, magazine regarding cricket and weather etc.

The first thing concerning the motion of an object is its kinematics. Kinematics is the study of motion of an object without discussing the cause of motion. In this unit, we will study the types of motion, scalar and vector quantities, the relation between displacement, speed, velocity and acceleration; linear motion and equations of motion.

## 2.1 REST AND MOTION

We see various things around us. Some of them are at rest while others are in motion.

A body is said to be at rest, if it does not change its position with respect to its surroundings.

Surroundings are the places in its neighbourhood where various objects are present. Similarly,

A body is said to be in motion, if it changes its position with respect to its surroundings.

**Figure 2.1:** The passengers in the bus are also moving with it.

The state of rest or motion of a body is relative. For example, a passenger sitting in a moving bus is at rest because he/she is not changing his/her position with respect to other passengers or objects in the bus. But to an observer outside the bus, the passengers and the objects inside the bus are in motion.


**Figure 2.:** A car and an aeroplane moving along a straight line are in linear motion.

## 2.2 TYPES OF MOTION

If we observe carefully, we will find that everything in the universe is in motion. However, different objects move differently. Some objects move along a straight line, some move in a curved path, and some move in some other way. There are three types of motion.

(I) Translatory motion (linear, random and circular)

(ii) Rotatory motion

(iii) Vibratory motion (to and fro motion)

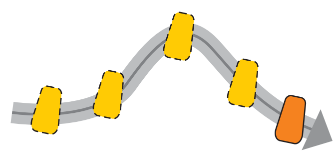


**Figure 2.3:** Translatory motion of an object along a curved path.

TRANSLATORY MOTION

Watch how various objects are moving. Do they move along a straight line? Do they move along a circle? A car moving in a straight line has translational motion. Similarly, an aeroplane moving straight is in translational motion.

In translational motion, a body moves along a line without any rotation. The line may be straight or curved.

**Figure 2.4:** Translatory motion of riders in Ferris wheel.

The object as shown in figure 2.3 moves along a curved path without rotation. This is the translational motion of the object. Riders moving in a Ferris wheel such as shown in figure 2.4 are also in translational motion. Their motion is in a circle without rotation. Translatory motions can be divided into linear motion, circular motion and random motion.


**Figure 2.5:** Linear motion of the ball falling down.

## LINEAR MOTION

We come across many objects which are moving in a straight line. The motion of objects such as a car moving on a straight and level road is linear motion.

**Straight line motion of a body is known as its linear motion.**

Aeroplanes flying straight in air and objects falling vertically down are also the examples of linear motion.

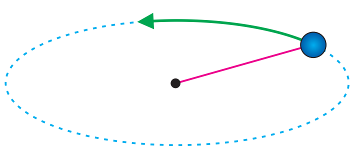


**Figure 2.6:** A stone tied at the end of a string moves in a circle.

## CIRCULAR MOTION

A stone tied at the end of a string can be made to whirl. What type of path is followed by the stone? The stone as shown in figure 2.6, moves in a circle and thus has circular motion.

**The motion of an object in a circular path is known as circular motion.**


**Figure 2.7:** A toy train moving on a circular track.

Figure 2.7 shows a toy train moving on a circular track. A bicycle or a car moving along a circular track possesses circular motion. Motion of the Earth around the Sun and motion of the moon around the Earth are also the examples of circular motions.

## RANDOM MOTION

Have you noticed the type of motion of insects and birds? Their movements are irregular.

**The disordered or irregular motion of an object is called random motion.**

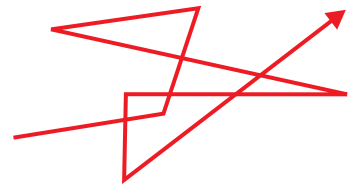


**Figure 2.8:** Random motion of gas molecules is called Brownian motion.

Thus, motion of insects and birds is random motion. The motion of dust or smoke particles in the air is also random motion. The Brownian motion of a gas or liquid molecules along a zig-zag path such as shown in figure 2.8 is also an example of random motion.

## ROTATORY MOTION

Study the motion of a top. It is spinning about an axis. Particles of the spinning top move in circles and thus individual particles possess circular motion. Does the top possess circular motion?

The top shown in figure 2.9 spins about its **axis** passing through it and thus it possesses rotatory motion. An **axis** is a line around which a body rotates. In circular motion, the point about which a body goes around, is outside the body. In rotatory motion, the line, around which a body moves about, is passing through the body itself.

Can you spin a ball at the tip of the finger?

**The spinning motion of a body about its axis is called its rotatory motion.**

**Figure 2.9:** Rotatory motion

Can you point out some more differences in circular and rotatory motion?

The motion of a wheel about its axis and that of a steering wheel are the examples of rotatory motion. The motion of the Earth around the Sun is circular motion and not the spinning motion. However, the motion of the Earth about its geographic axis that causes day and night is rotatory motion. Think of some more examples of rotatory motion.

**Figure2.10:** Vibratory motion of a child and a swing..

**Figure2.11:** Vibratory motion of the pendulum of a clock.

## VIBRATORY MOTION

Consider a baby in a swing as shown in figure 2.10. As it is pushed, the swing moves back and forth about its mean position. The motion of the baby repeats from one extreme to the other extreme with the swing.

**To and fro motion of a body about its mean position is known as vibratory motion**.

Figure 2.11 shows to and fro motion of the pendulum of a clock about its mean position, it is called vibratory motion. We can find many examples of vibratory motion around us. Look at the children in a see-saw as shown in figure 2.12. How the children move as they play the see-saw game? Do they possess vibratory motion as they move the see-saw?

**Figure 2.12**: Vibratory motion of children in a see-saw.

A baby in a cradle moving to and fro, to and fro motion of the hammer of a ringing electric bell and the motion of the string of a sitar are some of the examples of vibratory motion.

### Mini Exercise

1. When a body is said to be at rest?
2. Give an example of a body that is at rest and is in motion at the same time.
3. Mention the type of motion in each of the following:
   (i) A ball moving vertically upward.
   (ii) A child moving down a slide.
   (iii) Movement of a player in a football ground.
   (iv) The flight of a butterfly.
   (v) An athlete running in a circular track.
   (vi) The motion of a wheel.
   (vii) The motion of a cradle.

## 2.3 SCALARS AND VECTORS

In Physics, we come across various quantities such as mass, length, volume, density, speed and force etc. We divide them into scalars and vectors.

### SCALARS

A physical quantity which can be completely described by its magnitude is called a scalar. The magnitude of a quantity means its numerical value with an appropriate unit such as 2.5 kg, 40 s, 1.8 m, etc. Examples of scalars are mass, length, time, speed, volume, work and energy.

**A scalar quantity is described completely by its magnitude only.**

### VECTORS

A vector can be described completely by magnitude alongwith its direction. Examples of vectors are velocity, displacement, force, momentum, torque, etc. It would be meaningless to describe vectors without direction. For example, distance of a place from reference point is insufficient to locate that place. The direction of that place from reference point is also necessary to locate it.

**A vector quantity is described completely by magnitude and direction.**

Consider a table as shown in figure 2.13 (a). Two forces $F_1$ and $F_2$ are acting on it. Does it make any difference if the two forces act in opposite direction such as indicated in figure 2.13(b)?

Certainly the two situations differ from each other. They differ due to the direction of the forces acting on the table. Thus the description of a force would be incomplete if direction is not given. Similarly when we say, we are walking at the rate of 3 $kmh^{-1}$ towards north then we are talking about a vector.

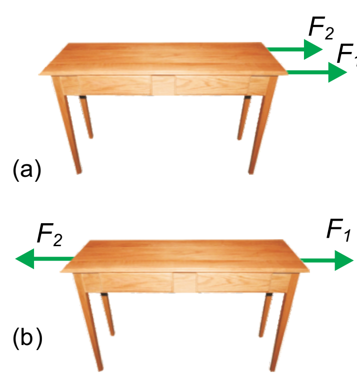



**Figure 2.13:** Two forces $F_1$ and $F_2$ (a) both acting in the same direction. (b) acting in opposite directions.

To differentiate a vector from a scalar quantity, we generally use bold letters to represent vector quantities, such as **F, a, d** or a bar or arrow over theirsymbols such as $\overline{F}$, $\overline{a}$, $\overline{d}$ or $\vec{F}$, $\vec{a}$ and $\vec{d}$.

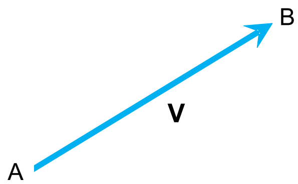



**Figure 2.14:** Graphical representation of a vector **V**

Graphically, a vector can be represented by a line segment with an arrow head. In figure 2.14, the line AB with arrow head at B represents a vector **V**. The length of the line AB gives the magnitude of the vector **V** on a selected scale. While the direction of the line from A to B gives the direction of the vector **V**.

### EXAMPLE 2.1

Represent a force of 80 N acting toward North of East.

### SOLUTION

Stepl: Draw two lines perpendicular to each other. Horizontal line represents East-West and vertical line represents North-South direction as shown in figure 2.15.

Step2: Select a suitable scale to represent the given vector. In this case we may take a scale which represents 20 N by 1 cm line.

Step3: Draw a line according to the scale in the direction of the vector. In this case, draw a line OA of length 4 cm along North-East.

Step4: Put an arrow head at the end of the line. In this case arrow head is at point A. Thus, the line OA will represent a vector i.e., the force of 80 N acting towards North-East.

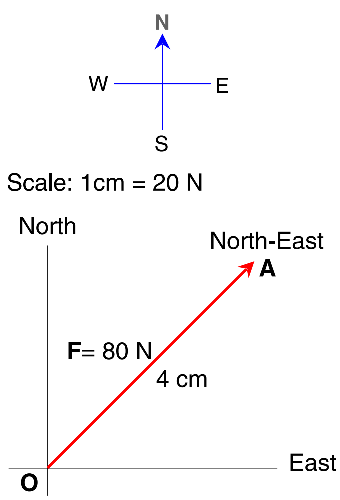



**Figure 2.15:** Representing 80 N force acting North-East.

## 2.4 TERMS ASSOCIATED WITH MOTION

When dealing with motion, we come across various terms such as the position of an object; the distance covered by it, its speed and so on. Let us explain some of the terms used.

## POSITION

The term position describes the location of a place or a point with respect to some reference point called origin. For example, you want to describe the position of your school from your home. Let the school be represented by S and home by H. The position of your school from your home will be represented by a straight line HS in the direction from H to S as shown in figure 2.16.

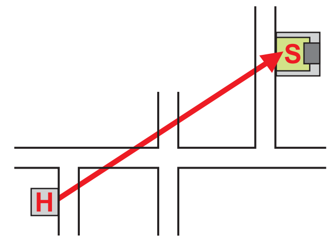



**Figure 2.16:** Position of the school S from the home H.

### DISTANCE AND DISPLACEMENT

Figure 2.17 shows a curved path. Let *S* be the length of the curved path between two points A and B on it. Then S is the distance between points A and B.

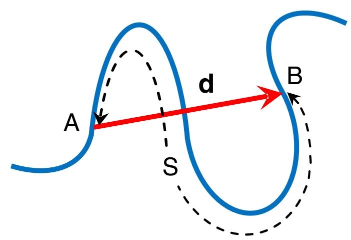



**Figure 2.17:** Distance S (dotted line) and displacement **d** (red line) from points A to B.

**Length of a path between two points is called the distance between those points.**

Consider a body that moves from point A to point B along the curved path. Join points A and B by a straight line. The straight line AB gives the distance which is the shortest between A and B. This shortest distance has magnitude d and direction from point A to B. This shortest distance d in a particular direction is called displacement. It is a vector quantity and is represented by **d.**

**Displacement is the shortest distance between two points which has magnitude and direction.**

### SPEED AND VELOCITY

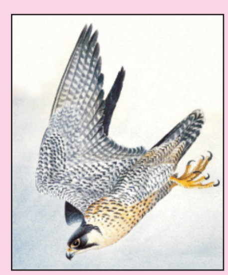
What information do we get by knowing the speed of a moving object?

Speed of an object is the rate at which it is moving. In other words, the distance moved by an object in unit time is its speed. This unit time may be a second, an hour, a day or a year.

**The distance covered by an object in unit time is called its speed.**

**DO YOU KNOW?**

Which is the fastest animal on the Earth?



Falcon can fly at a speed of 200 $kmh^{-1}$

Cheetah can run at a speed of 70 kmh"1.

$$\text{Speed} = \frac{\text{distance covered}}{\text{time taken}}$$

Distance = speed x time

$$\text{or} \qquad S = v\,t \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.1)$$

Here S is the distance covered by the object, $v$ is its speed and $t$ is the time taken by it. Distance is a scalar; therefore, speed is also a scalar. SI unit of speed is metre per second ($ms^{-1}$).

## UNIFORM SPEED

In equation 2.**1,** $v$ is the average speed of a body during time f. It is because the speed of the body may be changing during the time interval $t$. However, if the speed of a body does not vary and has the same value then the body is said to possess uniform speed.

**A body has uniform speed if it covers equal distances in equal intervals of time however short the interval may be.**

### DO YOU KNOW?

A motorway speed camera

A LIDAR gun is light detection and ranging speed gun. It uses the time taken by laser pulse to make a series of measurements of a vehicle's distance from the gun. The data is then used to calculate the vehicle's speed.

## VELOCITY

The velocity tells us not only the speed of a body but also the direction along which the body is moving. Velocity of a body is a vector quantity. It is equal to the displacement of a body in unit time.

**The rate of displacement of a body is called its velocity.**

$$\text{Velocity} = \frac{\text{displacement}}{\text{time taken}}$$

$$\text{or} \qquad \mathbf{v} = \frac{\mathbf{d}}{t}$$

$$\mathbf{d} = \mathbf{v}t \quad \ldots \quad \ldots \quad \ldots \quad \ldots (2.2)$$

Here **d** is the displacement of the body moving with velocity **v** in time $t$. SI unit of velocity is the same as speed i.e., metre per second ($ms^{-1}$).

### DO YOU KNOW?

A paratrooper attains a uniform velocity called terminal velocity with which it comes to ground,

## UNIFORM VELOCITY

In equation 2.2, **v** is the average velocity of a body during time $t$. It is because the velocity of the body may be changing during the time interval *t*. However, in many cases the speed and direction of a body does not change. In such a case the body possesses uniform velocity. That is the velocity of a body during any interval of time has the same magnitude and direction. Thus

**A body has uniform velocity if it covers equal displacement in equal intervals of time however short the interval may be.**

## EXAMPLE 2.2

A sprinter completes its 100 metre race in 12s. Find its average speed.

## SOLUTION

Total distance = 100 m

Total time taken = 12 s

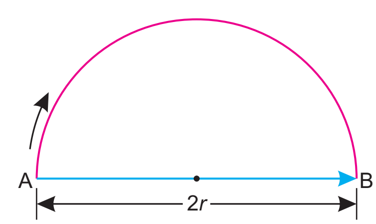
$$\text{Average speed} = \frac{\text{Total distance moved}}{\text{Total time taken}}$$

$$= \frac{100\,\text{m}}{12\,\text{s}} = 8.33\ \text{ms}^{-1}$$

Thus the speed of the sprinter is 8.33 $\text{ms}^{-1}$ .

## EXAMPLE 2.3

A cyclist completes half round of a circular track of radius 318 m in 1.5 minutes. Find its speed and velocity.




Radius of track $r$ = 318 m

Time taken $t$ = 1 min. 30 s = 90 s

Distance covered = $\pi \times$ radius

= $3.14 \times 318$ m = 999 m

Displacement = $2\,r$

= $2 \times 318$ m = 636 m

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{999\,\text{m}}{90\,\text{s}} = 11.1\ \text{ms}^{-1}$$

$$\text{velocity} = \frac{\text{displacement}}{\text{time taken}}$$

$$= \frac{636\text{ m}}{90\text{ s}} = 7.07\text{ ms}^{-1}$$

Thus speed of the cyclist is 11.1 $ms^{-1}$ along the track and its velocity is about 7.1 $ms^{-1}$ along the diameter AB of the track.

## ACCELERATION

When does a body possess acceleration?

In many cases the velocity of a body changes due to a change either in its magnitude or direction or both. The change in the velocity of a body causes acceleration in it.

**Acceleration is defined as the rate of change of velocity of a body.**

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

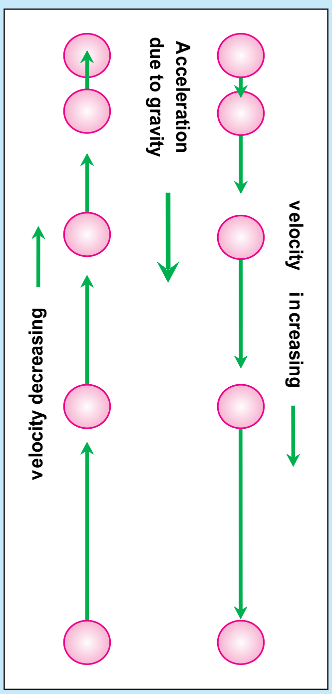
$$\text{Acceleration} = \frac{\text{final velocity - initial velocity}}{\text{time taken}}$$

$$\mathbf{a} = \frac{\mathbf{v_f} - \mathbf{v_i}}{t} \quad \ldots \quad \ldots \quad \ldots (2.3)$$

Taking acceleration as a, initial velocity as $V_i$, final velocity as $v_f$ and t is the time interval. SI unit of acceleration is metre per second per second ($ms^{-2}$).

### USEFUL INFORMATION

Acceleration of a moving object is in the direction of velocity if its velocity is increasing. Acceleration of the object is opposite to the direction of velocity if its velocity is decreasing.




## UNIFORM ACCELERATION

The average acceleration of a body given by equation 2.3 is **a** during time t. Let the time t is divided into many smaller intervals of time. If the rate of change of velocity during all these intervals remains constant then the acceleration **a** also remains constant. Such a body is said to possess uniform acceleration.

**A body has uniform acceleration if it has equal changes in velocity in equal intervals of time however short the interval may be.**

Acceleration of a body is positive if its velocity increases with time. The direction of this acceleration is the same in which the body is moving without change in its direction. Acceleration of a body is negative if velocity of the body decreases. The direction of negative acceleration is opposite to the direction in which the body is moving. Negative acceleration is also called **deceleration** or **retardation.**

## EXAMPLE 2.4

A car starts from rest. Its velocity becomes 20 ms in 8 s. Find its acceleration.

## SOLUTION

Initial velocity $v_i$ = $0\ \text{ms}^{-1}$

Final velocity $v_f$ = $20\ \text{ms}^{-1}$

Time taken $t$ = 8 s

as $$a = \frac{v_f - v_i}{t}$$

or $$a = \frac{20\ \text{ms}^{-1} - 0\ \text{ms}^{-1}}{8\ \text{s}}$$

$$= 2.5\ \text{ms}^{-2}$$

Thus the acceleration of the car is $2.5\ \text{ms}^{-2}$

## EXAMPLE 2.5

Find the retardation produced when a car moving at a velocity of $30\ \text{ms}^{-1}$ slows down uniformly to $15\ \text{ms}^{-1}$ in 5s.

## SOLUTION

Initial velocity $v_i$ = $30\ \text{ms}^{-1}$

Final velocity $v_f$ = $15\ \text{ms}^{-1}$

Change in velocity = $v_f - v_i$

= $15\ \text{ms}^{-1} - 30\ \text{ms}^{-1}$

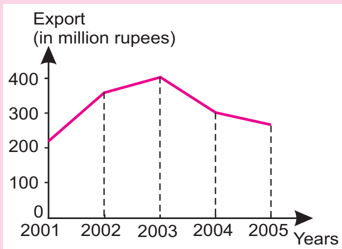
= $-15\ \text{ms}^{-1}$

Time taken $t$ = 5 s

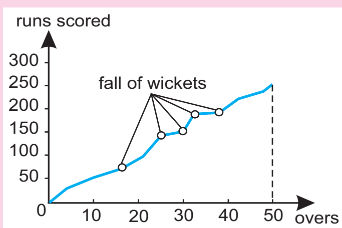
$a$ = ?

## DO YOU KNOW?

A graph may also be used in everyday life such as to show year-wise growth/decline of export, month-wise rainfall, a patient's temperature record or runs per over scored by a team and so on.




Export of rice from 2001-2005




Runs scored by a cricket team.

as Acceleration $= \dfrac{\text{change in velocity}}{\text{time interval}}$

or $$a = \frac{-15\ \text{ms}^{-1}}{5\ \text{s}} = -3\ \text{ms}^{-2}$$

Since negative acceleration is called as deceleration. Thus deceleration of the car is $3\ \text{ms}^{-2}$.

## 2.5 GRAPHICAL ANALYSIS OF MOTION

Graph is a pictorial way of presenting information about the relation between various quantities. The quantities between which a graph is plotted are called the variables. One of the quantities is called the independent quantity and the other quantity, the value of which varies with the independent quantity is called the dependent quantity.

### DISTANCE-TIME GRAPH

It is useful to represent the motion of objects using graphs. The terms distance and displacement are used interchangeably when the motion is in a straight line. Similarly if the motion is in a straight line then speed and velocity are also used interchangeably. In a distance-time graph, time is taken along horizontal axis while vertical axis shows the distance covered by the object.

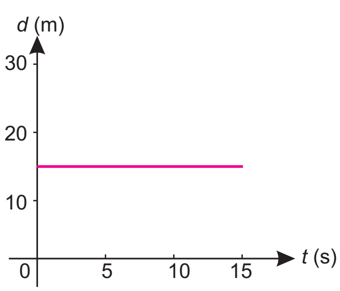



**Figure 2.18:** Distance-time graph when the object is at rest.

### OBJECT AT REST

In the graph shown in figure 2.18, the distance moved by the object with time is zero. That is, the object is at rest. Thus a horizontal line parallel to time axis on a distance-time graph shows that speed of the object is zero.

### OBJECT MOVING WITH CONSTANT SPEED

The speed of an object is said to be constant if it covers equal distances in equal intervals of time. The distance-time graph as shown in figure 2.19 is a straight line. Its slope gives the speed of the object. Consider two points A and B on the graph

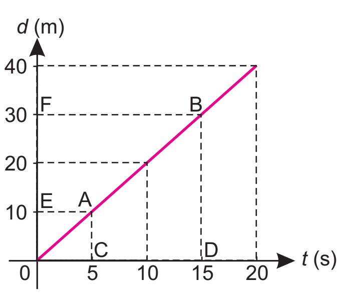



**Figure 2.19:** Distance time graph showing constant speed.

Speed of the object $= $ slope of line AB

$$= \frac{\text{distance EF}}{\text{time CD}}$$

$$= \frac{20\text{ m}}{10\text{ s}} = 2\text{ ms}^{-1}$$

The speed found from the graph is 2 $m^{-1}$

## OBJECT MOVING WITH VARIABLE SPEED

When an object does not cover equal distances in equal intervals of time then its speed is not constant. In this case the distance-time graph is not a straight line as shown in figure 2.20. The slope of the curve at any point can be found from the slope of the tangent at that point. For example,

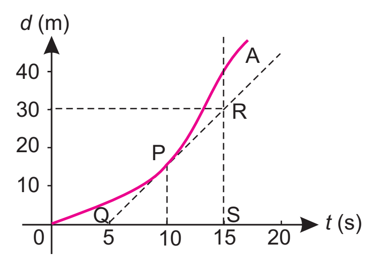



**Figure 2.20:** Distance- time graph showing variable speed.

Slope of the tangent at P $= \frac{RS}{QS}$

$$= \frac{30\text{ m}}{10\text{ s}} = 3\text{ ms}^{-1}$$

Thus, speed of the object at P is 3 $ms^{-1}$. The speed is higher at instants when slope is greater; speed is zero at instants when slope is horizontal.

## EXAMPLE 2.6

Figure 2.21 shows the distance-time graph of a moving car. From the graph, find

(a) the distance car has traveled.
(b) the speed during the first five seconds.
(c) average speed of the car.
(d) speed during the last 5 seconds.

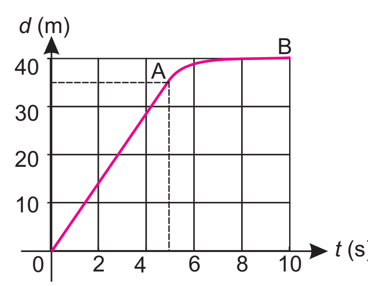



**Figure 2.21:** Distance-time graph of a car in example 2.6

## SOLUTION

(a) Total distance travelled $= 40$ m
(b) Distance travelled during first 5s is 35 m

$\therefore$ Speed $= \frac{35\text{ m}}{5\text{ s}}$

$= 7\text{ ms}^{-1}$

(c) Average speed $= \frac{40\text{ m}}{10\text{ s}}$

$= 4\text{ ms}^{-1}$

(d) Distance moved during the last 5s = 5 m

$$\therefore \quad \text{Speed} = \frac{5\text{ m}}{5\text{ s}} = 1\text{ ms}^{-1}$$

## SPEED-TIME GRAPH

In a speed-time graph, time is taken along x-axis and speed is taken along y-axis.

## OBJECT MOVING WITH CONSTANT SPEED

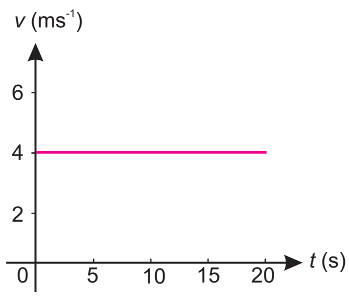



**Figure 2.22**: Speed-time graph showing constant speed.

When the speed of an object is constant (4 $\text{ms}^{-1}$) with time, then the speed-time graph will be a horizontal line parallel to time-axis along x-axis as shown in figure 2.22. In other words, a straight line parallel to time axis represents constant speed of the object.

## OBJECT MOVING WITH UNIFORMLY CHANGING SPEED (uniform acceleration)

Let the speed of an object be changing uniformly. In such a case speed is changing at constant rate. Thus its speed-time graph would be a straight line such as shown in figure 2.23. A straight line means that the object is moving with uniform acceleration. Slope of the line gives the magnitude of its acceleration.

## EXAMPLE 2.7

Find the acceleration from speed-time graph shown in figure 2.23.

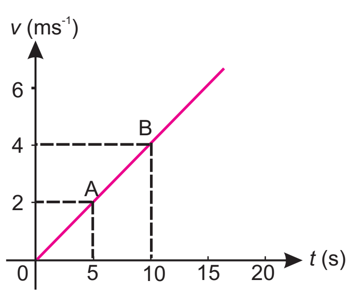



**Figure 2.23**: Graph of an object moving with unifrm acceleration.

## SOLUTION

On the graph in figure 2.23, point A gives speed of the object as 2 $\text{ms}^{-1}$ after 5 s and point B gives speed of the object as 4 $\text{ms}^{-1}$ after 10 s

as acceleration = slope of AB

where slope = change in velocity/time interval

$$\therefore \quad \text{acceleration} = \frac{4\text{ ms}^{-1} - 2\text{ ms}^{-1}}{10\text{ s} - 5\text{ s}}$$

$$= \frac{2\text{ ms}^{-1}}{5\text{ s}} = 0.4\text{ ms}^{-2}$$

Speed-time graph in figure 2.23 gives acceleration of the object as $0.4\ ms^{-2}$.

## EXAMPLE 2.8

Find the acceleration from speed-time graph shown in figure 2.24.

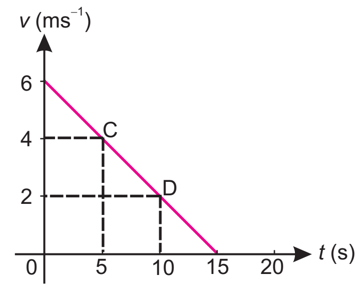



**Figure 2.24**: Graph of an object moving with uniform deceleration.

## SOLUTION

The graph in figure 2.24 shows that the speed of the object is decreasing with time. The speed after 5s is $4\ ms^{-1}$ and it becomes $2\ ms^{-1}$ after 10 s.
as acceleration = slope of CD

$$= \frac{2\ ms^{-1} - 4\ ms^{-1}}{10\ s - 5\ s}$$

$$= -\frac{2\ ms^{-1}}{5\ s} \qquad = -0.4\ ms^{-2}$$

Speed-time graph in figure 2.24 gives negative slope. Thus, the object has deceleration of $0.4\ ms^{-2}$.

## DISTANCE TRAVELLED BY A MOVING OBJECT

The area under a speed-time graph represents the distance travelled by the object. If the motion is uniform then the area can be calculated using appropriate formula for geometrical shapes represented by the graph.

## EXAMPLE 2.9

A car moves in a straight line. The speed-time graph of its motion is shown in figure 2.25.

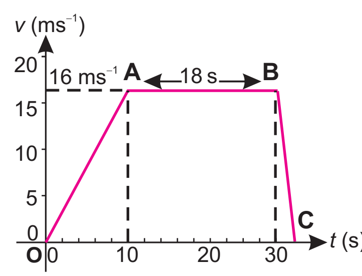



**Figure 2.25**: Speed time graph of a car during 30 seconds.

From the graph, find

(a) Its acceleration during the first 10 seconds.

(b) Its deceleration during the last 2 seconds.

(c) Total distance travelled.

(d) Average speed of the car during its journey.

**SOLUTION**

(a) Acceleration during the first 10 seconds

$$= \frac{\text{change in velocity}}{\text{time taken}}$$

$$= \frac{16\ \text{ms}^{-1} - 0\ \text{ms}^{-1}}{10\ \text{s}}$$

$$= 1.6\ \text{ms}^{-2}$$

(b) Acceleration during the last 2 seconds

$$= \frac{0\ \text{ms}^{-1} - 16\ \text{ms}^{-1}}{2\ \text{s}}$$

$$= -8\ \text{ms}^{-2}$$

(c) Total distance travelled

= area under the graph (trapezium OABC)

$$= \frac{1}{2}(\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2}(18\ \text{s} + 30\ \text{s}) \times (16\ \text{ms}^{-1})$$

$$= \frac{1}{2}(48\ \text{s}) \times (16\ \text{ms}^{-1})$$

$$= 384\ \text{m}$$

(d) Average speed $= \dfrac{\text{Total distance covered}}{\text{Time taken}}$

$$= \frac{384\ \text{m}}{30\ \text{s}} = 12.8\ \text{ms}^{-1}$$

## 2.6 EQUATIONS OF MOTION

There are three basic equations of motion for bodies moving with uniform acceleration. These equations relate initial velocity, final velocity, acceleration, time and distance covered by a moving body. To simplify the derivation of these equations, we assume that the motion is along a straight line. Hence, we consider only the magnitude of displacements, velocities, and acceleration.

Consider a body moving with initial velocity $v_i$ in a straight line with uniform acceleration a. Its velocity becomes $v_f$ after time *t*. The motion of body is described by speed-time graph as shown in figure 2.26 by line AB. The slope of line AB is acceleration a. The total distance covered by the body is shown by the shaded area under the line AB. Equations of motion can be obtained easily from this graph.

## FIRST EQUATION OF MOTION

Speed-time graph for the motion of a body is shown in figure 2.26. Slope of line AB gives the acceleration a of a body.

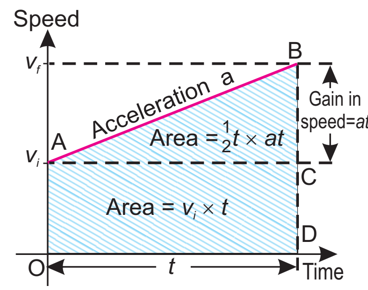



**Figure 2.26:** Speed-time graph. Area under the graph gives the distance covered by the body.

$$\text{Slope of line AB} = a = \frac{BC}{AC} = \frac{BD - CD}{OD}$$

as $BD = v_f$, $CD = v_i$ and $OD = t$

$$\text{Hence} \quad a = \frac{v_f - v_i}{t}$$

$$\text{or} \quad v_f - v_i = at \quad \ldots \ldots \ldots \ldots (2.4)$$

$$\therefore \quad v_f = v_i + at \quad \ldots \ldots \ldots (2.5)$$

## SECOND EQUATION OF MOTION

In speed-time graph shown in figure 2.26, the total distance S travelled by the body is equal to the total area OABD under the graph. That is

Total distance S= area of (rectangle OACD + triangle ABC)

*Area of rectangle OACD* $= OA \times OD$

$= v_i \times t$

*Area of triangle ABC* $= \frac{1}{2}(AC \times BC)$

$= \frac{1}{2} t \times at$

Since Total area OABD = area of rectangle OACD + area of triangle ABC

Putting values in the above equation, we get

$$S = v_i t + \frac{1}{2} t \times at$$

$$S = v_i t + \frac{1}{2} at^2 \ \ldots \ \ldots \ \ldots \ \ldots \ \ldots \ (2.6)$$

## THIRD EQUATION OF MOTION

In speed-time graph shown in figure 2.26, the total distance S travelled by the body is given by the total area OABD under the graph.

$$\text{Total area OABD} = S = \frac{\text{OA} + \text{BD}}{2} \times \text{OD}$$

$$\text{or} \quad 2S = (\text{OA} + \text{BD}) \times \text{OD}$$

Multiply both sides by $\frac{\text{BC}}{\text{OD}}$, we get: $\left(\because \frac{\text{BC}}{\text{OD}} = a\right)$

$$2S \times \frac{\text{BC}}{\text{OD}} = (\text{OA} + \text{BD}) \times \text{OD} \times \frac{\text{BC}}{\text{OD}}$$

$$2S \times \frac{\text{BC}}{\text{OD}} = (\text{OA} + \text{BD}) \times \text{BC} \quad \ldots \quad (2.7)$$

Putting the values in the above equation 2.7, we get

$$2S \times a = (v_i + v_f) \times (v_f - v_i)$$

$$2aS = v_f^2 - v_i^2 \quad \ldots \quad \ldots \quad \ldots \quad (2.8)$$

## EXAMPLE 2.10

A car travelling at 10 $ms^{-1}$ accelerates uniformly at 2 $ms^{-2}$. Calculate its velocity after 5 s.

## SOLUTION

$v_i$ = 10 $ms^{-1}$

$a$ = 2 $ms^{-2}$

$t$ = 5s

$v_f$ = ?

Using the equation (2.5), we get

$$v_f = v_i + at$$

$$v_f = 10\ ms^{-1} + 2\ ms^{-2} \times 5\ s$$

or $v_f = 20\ \text{ms}^{-1}$

The velocity of the car after 5 s is 20 ms

**EXAMPLE 2.11**

A train slows down from 80 $\text{kmh}^{-1}$ with a uniform retardation of 2 $\text{ms}^{-2}$ . How long will it take to attain a speed of 20 $\text{kmh}^{-1}$?

**SOLUTION**

$$v_i = 80\ \text{kmh}^{-1}$$
$$= \frac{80 \times 1000\ \text{m}}{60 \times 60\ \text{s}}$$
$$= 22.2\ \text{ms}^{-1}$$
$$v_f = 20\ \text{kmh}^{-1}$$
$$= \frac{20 \times 1000\ \text{m}}{60 \times 60\ \text{s}}$$
$$= 5.6\ \text{ms}^{-1}$$
$$a = -2\ \text{ms}^{-1}$$
$$t = ?$$

Using equation 2.4, we get

$$v_f = v_i + at$$

or
$$t = \frac{v_f - v_i}{a}$$
$$\frac{5.6\ \text{ms}^{-1} - 22.2\ \text{ms}^{-1}}{-2\ \text{ms}^{-2}}$$

or
$$t = 8.3\ \text{s}$$

Thus the train will take 8.3 s to attain the required speed.

**EXAMPLE 2.12**

A bicycle accelerates at 1 $\text{ms}^{-2}$ from an initial velocity of 4 $\text{ms}^{-1}$ for 10 s. Find the distance moved by it during this interval of time.

**SOLUTION**

$$v_i = 4\ \text{ms}^{-1}$$
$$a = 1\ \text{ms}^{-2}$$
$$t = 10\ \text{s}$$
$$S = ?$$

Applying equation (2.6), we get

**USEFUL INFORMATION**

- **To convert $\text{ms}^{-1}$ to $\text{kmh}^{-1}$**

$1\ \text{ms}^{-1} = 0.001\ \text{km} \times 3600\ \text{h}^{-1}$

$= 3.6\ \text{kmh}^{-1}$

The Multiply acceleration in $\text{ms}^{-1}$ by 3.6 to get speed in $\text{kmh}^{-1}$ e.g.

$20\ \text{ms}^{-1} = 20 \times 3.6\ \text{kmh}^{-1}$

$= 72\ \text{kmh}^{-1}$

- **To convert $\text{kmh}^{-1}$ to $\text{ms}^{-1}$**

$$1\ \text{kmh}^{-1} = \frac{1000\ \text{m}}{60 \times 60\ \text{s}} = \frac{10}{36}\ \text{ms}^{-1}$$

Thus multiply speed in $\text{kmh}^{-1}$ by $\frac{10}{36}$ to get speed in $\text{ms}^{-1}$ e.g.,

$$50\ \text{kmh}^{-1} = 50 \times \frac{10}{36}\ \text{ms}^{-1}$$
$$= 13.88\ \text{ms}^{-1}$$

Similarly

- **To convert $\text{ms}^{-2}$ to $\text{kmh}^{-2}$**

Multiply acceleration in $\text{ms}^{-2}$ by {(3600x3600)/1000}= 12960 to get its value in $\text{kmh}^{-2}$.

- **To convert $\text{kmh}^{-2}$ to $\text{ms}^{-2}$**

Divide acceleration in $\text{kmh}^{-2}$ by 12960 to get its value in $\text{ms}^{-2}$.

$$S = v_i t + \tfrac{1}{2} at^2$$

$$S = 4\ \text{ms}^{-1} \times 10\ \text{s} + \tfrac{1}{2} \times 1\ \text{ms}^{-2} \times (10\ \text{s})^2$$

$$\text{or} \quad S = 40\ \text{m} + 50\ \text{m} = 90\ \text{m}$$

Thus, the bicycle will move 90 metres in 10 seconds.

## EXAMPLE 2.13

A car travels with a velocity of 5 $\text{ms}^{-1}$. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 15 $\text{ms}^{-1}$, find the acceleration and the time to travel this distance.

## SOLUTION

$$v_i = 5\ \text{ms}^{-1}$$

$$S = 50\ \text{m}$$

$$v_f = 15\ \text{ms}^{-1}$$

$$a = ?$$

$$t = ?$$

Putting values in the third equation of motion, we get

$$2aS = v_f^2 - v_i^2$$

$$\therefore \quad 2a \times 50\ \text{m} = (15\ \text{ms}^{-1})^2 - (5\ \text{ms}^{-1})^2$$

$$(100\ \text{m})\, a = (225 - 25)\ \text{m}^2\text{s}^{-2}$$

$$a = \frac{200\ \text{m}^2\text{s}^{-2}}{100\ \text{m}}$$

$$\text{or} \quad a = 2\ \text{ms}^{-2}$$

Using first equation of motion to find $t$, we get

$$v_f = v_i + at$$

$$\therefore \quad 15\ \text{ms}^{-1} = 5\ \text{ms}^{-1} + 2\ \text{ms}^{-2} \times t$$

$$15\ \text{ms}^{-1} - 5\ \text{ms}^{-1} = 2\ \text{ms}^{-2} \times t$$

$$\text{or} \quad 2\ \text{ms}^{-2} \times t = 10\ \text{ms}^{-1}$$

$$\text{or} \quad t = \frac{10\ \text{ms}^{-1}}{2\ \text{ms}^{-2}}$$

$$= 5\ \text{s}$$

Thus, the acceleration of the car is 2 $\text{ms}^{-2}$ and it takes 5 seconds to travel 50 m distance.

## 2.7 MOTION OF FREELY FALLING BODIES

Drop an object from some height and observe its motion. Does its velocity increase, decrease or remain constant as it approaches the ground?

Galileo was the first scientist to notice that all the freely falling objects have the same acceleration independent of their masses. He dropped various objects of different masses from the leaning tower of Pisa. He found that all of them reach the ground at the same time. The acceleration of freely falling bodies is called **gravitational acceleration.** It is denoted by ***g.*** On the surface of the Earth, its value is approximately 10 $ms^{-2}$. For bodies falling down freely ***g*** is positive and is negative for bodies moving up.

**Figure 2.27:** Learning Tower of Pisa

### EQUATIONS OF MOTION FOR BODIES MOVING UNDER GRAVITY

$$v_f = v_i + gt$$

$$h = v_i t + \frac{1}{2} gt^2$$

$$2gh = v_f^2 - v_i^2$$

### EXAMPLE 2.14

A stone is dropped from the top of a tower. The stone hits the ground after 5 seconds. Find

- the height of the tower.
- the velocity with which the stone hits the ground.

### SOLUTION

Initial velocity v, = 0

Gravitational acceleration g = 10 $ms^{-2}$

$t$ = 5 s

$S = h = ?$

$v_f = ?$

(a) Applying the equation

$$h = v_i t + \tfrac{1}{2} g t^2, \text{ we get}$$

$$h = 0 \times 5\text{ s} + \tfrac{1}{2} \times 10\text{ ms}^{-2} \times (5\text{ s})^2$$

$$\text{or} \quad h = 0 + 125\text{ m}$$

$$\therefore \quad h = 125\text{ m}$$

(b) Applying the equation

$$v_f^2 - v_i^2 = 2gh$$

$$v_f^2 - (0)^2 = 2 \times 10\text{ ms}^{-2} \times 125\text{ m}$$

$$v_f^2 = 2500\text{ m}^2\text{s}^{-2}$$

$\therefore \quad v_f = 50\ ms^{-1}$

Thus the height of the tower is 125 metres and it will hit the ground with a velocity of 50 $ms^{-1}$.

EXAMPLE 2.15

A boy throws a ball vertically up. It returns to the ground after 5 seconds. Find

(a) the maximum height reached by the ball.

(b) the velocity with which the ball is thrown up.

SOLUTION

Initial velocity (upward) $\quad v_i = ?$

Gravitational acceleration $\quad g = -10\ ms^{-2}$

Time for up and down motion $\quad t_o = 5\ s$

Velocity at maximum height $\quad v_f = 0$

$S = h = ?$

As the acceleration due to gravity is uniform, hence the time t taken by the ball to go up will be equal to the time taken to come down = ½ $t_o$

or $\quad t = 1/2 \times 5\ s = 2.5\ s$

(b) applying the equation (2.5), we get

$$v_f = v_i + g\,t$$

$$0 = v_i - 10\ ms^{-2} \times 2.5\ s$$

$$= v_i - 25\ ms^{-1}$$

$$\therefore \quad v_i = 25\ ms^{-1}$$

(a) Applying the equation (2.6), we get

$$h = v_i t + 1/2\, g\, t^2$$

$$h = 25\ ms^{-1} \times 2.5\ s - 10\ ms^{-2} \times (2.5\ s)^2$$

$$\text{or} \quad h = 62.5\ m - 31.25\ m = 31.25\ m$$

Thus, the ball was thrown up with a speed of 25 $ms^{-1}$ and the maximum height to which the ball rises is 31.25 m.

# SUMMARY

- A body is said to be at rest, if it does not change its position with respect to its surroundings.
- A body is said to be in motion, if it changes its position with respect to its surroundings.
- Rest and motion are always relative. There is no such thing as absolute rest or absolute motion.
- Motion can be divided into the following three types.
    - Translatory motion: In which a body moves without any rotation.
    - Rotatory motion: In which a body spins about its axis.
    - Vibratory motion: In which a body moves to and fro about its mean position.
- Physical quantities which are completely described by their magnitude only are known as scalars.
- Physical quantities which are described by their magnitude and direction are called vectors.
- Position means the location of a certain place or object from a reference point.
- The shortest distance between two points is called the displacement.
- The distance travelled in any direction by a body in unit time is called speed.
- If the speed of a body does not change with time then its speed is uniform.
- Average speed of a body is the ratio of the total distance covered to the total time taken.
- We define velocity as rate of change of displacement or speed in a specific direction.
- Average velocity of a body is defined as the ratio of its net displacement to the total time.
- If a body covers equal displacements in equal intervals of time, however small the interval may be, then its velocity is said to be uniform.
- The rate of change of velocity of a body is called acceleration.
- A body has uniform acceleration if it has equal changes in its velocity in equal intervals of time, however small the interval may be.
- Graph is a pictorial way of describing information as to how various quantities are related to each other.
- Slope of the distance-time graph gives the speed of the body.
- Distance - time graphs provide useful information about the motion of an object. Slope of the displacement-time graph gives the velocity of the body.
- Distance covered by a body is equal to area under speed - time graph.
- Speed-time graph is also useful for studying motion along a straight line.
- The distance travelled by a body can also be found from the area under a velocity - time graph if the motion is along a straight line.

- Equations of motion for uniformly accelerated motion are:
  - $v_f = v_i + at$
  - $S = v_i t + \frac{1}{2} at^2$
  - $2aS = v_f^2 - v_i^2$

- When a body is dropped freely it falls down with an acceleration towards Earth. This acceleration is called acceleration due to gravity and is denoted by *g.* The numerical value of *g* is approximately 10 $ms^{-2}$ near the surface of the Earth.

# QUESTIONS

**2.1 Encircle the correct answer v. from the given choices:**

**i.** A body has translatory motion if it moves along a

(a) straight line
(b) circle
(c) line without rotation
(d) curved path

**ii.** The motion of a body about an VL axis is called

(a) circular motion
(b) rotatory motion
(c) vibratory motion
(d) random motion

**iii.** Which of the following is a vector quantity?

(a) speed (b) distance
(c) displacement (d) power

**iv.** If an object is moving with constant speed then its distance-time graph will be a straight line.

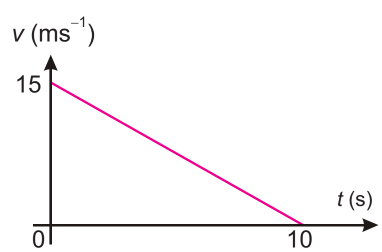
(a) along time-axis
(b) along distance-axis
(c) parallel to time-axis
(d) inclined to time-axis

**v.** A straight line parallel to time-axis on a distance-time graph tells that the object is

(a) moving with constant speed
(b) at rest
(c) moving with variable speed
(d) in motion

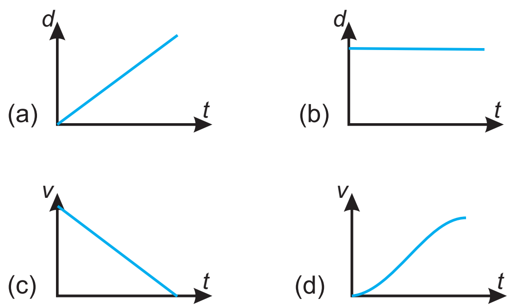
**vi.** The speed-time graph of a car is shown in the figure, which of the following statement is true?

(a) car has an acceleration of 1.5 $m^{-2}$
(b) car has constant speed of 7.5 $ms^{-1}$
(c) distance travelled by the car is 75 m
(d) average speed of the car is 15 $ms^{-1}$




**(vi)** Speed-time graph of a car.

**vii.** Which one of the following graphs is representing uniform acceleration?




**viii.** By dividing displacement of a moving body with time, we obtain

(a) speed (b) acceleration

(c) velocity (d) deceleration

**ix.** A ball is thrown vertically upward. Its velocity at the highest point is :

(a) -10 $ms^{-1}$ (b) zero

(c) 10 $ms^{-2}$ (d) none of these

**x.** A change in position is called:

(a) speed (b) velocity

(c) displacement (d) distance

**xi.** A train is moving at a speed of 36 $kmh^{-1}$. Its speed expressed in $ms^{-1}$ is:

(a) 10 $ms^{-1}$ (b) 20 $ms^{-1}$

(c) 25 $ms^{-1}$ (d) 30 $ms^{-1}$

**xii.** A car starts from rest. It acquires a speed of 25 $ms^{-1}$ after 20 s. The distance moved by the car during this time is:

(a) 31.25 m (b) 250 m

(c) 500 m (d) 5000 m

**2.2 Explain translatory motion and give examples of various types of translatory motion.**

**2.3 Differentiate between the following:**

(i) Rest and motion.

(ii) Circular motion and rotatory motion.

(iii) Distance and displacement

(iv) Speed and velocity.

(v) Linear and random motion.

(vi) Scalars and vectors.

**2.4 Define the terms speed, velocity, and acceleration.**

**2.5 Can a body moving at a constant speed have acceleration?**

**2.6 How do riders in a Ferris wheel possess translatory motion but not rotatory motion?**

**2.7 Sketch a distance-time graph for a body starting from rest. How will you determine the speed of a body from this graph?**

**2.8 What would be the shape of a speed - time graph of a body moving with variable speed?**

**2.9 Which of the following can be obtained from speed - time graph of a body?**

(i) Initial speed.

(ii) Final speed.

(iii) Distance covered in time *t.*

(iv) Acceleration of motion.

**2.10 How can vector quantities be represented graphically?**

**2.11 Why vector quantities cannot be added and subtracted like scalar quantities?**

**2.12** How are vector quantities important to us in our daily life?

**2.13** Derive equations of motion for uniformly accelerated rectilinear motion.

**2.14** Sketch a velocity - time graph for the motion of the body. From the graph explaining each step, calculate total distance covered by the body.

# PROBLEM

**2.1** A train moves with a uniform velocity of 36 $kmh^{-1}$ for 10 s. Find the distance travelled by it. (100 m)

**2.2** A train starts from rest. It moves through 1 km in 100 s with uniform acceleration. What will be its speed at the end of 100 s.

(20 $ms^{-1}$)

**2.3** A car has a velocity of 10 $ms^{-1}$. It accelerates at 0.2 $ms^{-2}$ for half minute. Find the distance travelled during this time and the final velocity of the car.

(390 m, 16 $ms^{-1}$)

**2.4** A tennis ball is hit vertically upward with a velocity of 30 $ms^{-1}$. It takes 3 s to reach the highest point. Calculate the maximum height reached by the ball. How long it will take to return to ground?

(45 m, 6 s)

**2.5** A car moves with uniform velocity of 40 ms"[1] for 5 s. It comes to rest in the next 10 s with uniform deceleration. Find (i) deceleration (ii) total distance travelled by the car.

(-4 $ms^{-2}$, 400 m)

**2.6** A train starts from rest with an acceleration of 0.5 $ms^{-2}$. Find its speed in $kmh^{-2}$, when it has moved through 100 m. (36 $kmh^{-2}$)

**2.7** A train staring from rest, accelerates uniformly and attains a velocity 48 $kmh^{-1}$ in 2 minutes. It travels at this speed for 5 minutes. Finally, it moves with uniform retardation and is stopped after 3 minutes. Find the total distance travelled by the train. (6000 m)

**2.8** A cricket ball is hit vertically upwards and returns to ground 6 s later. Calculate (i) maximum height reached by the ball, (ii) initial velocity of the ball.

(45 m, 30 $ms^{-1}$)

**2.9** When brakes are applied, the speed of a train decreases from 96 $kmh^{-1}$ to 48 $kmh^{-1}$ in 800 m. How much further will the train move before coming to rest? (Assuming the retardation to be constant). (266.66 m)

**2.10** In the above problem, find the time taken by the train to stop after the application of brakes.

(80 s)